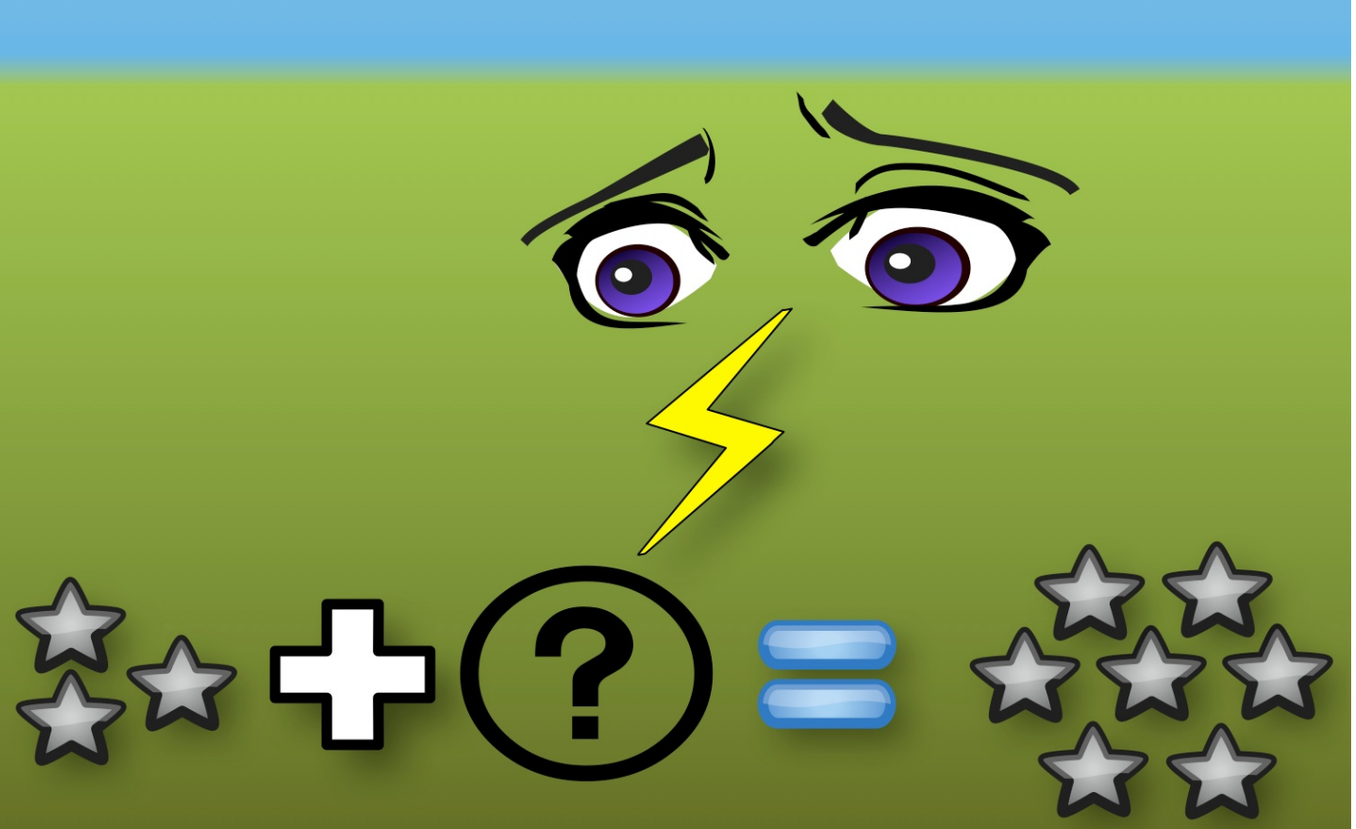
Mathematical Tale Winds Series:

ADDITION

Meaning & Problem Solving



J. A. Ameis, Ph.D. Mathematics Education

Chapter 1

What?

Why?

How?

Purpose

WHO IS THIS BOOK FOR?

- ☑ Home-schooling parents who desire well-designed activities for teaching meaningful mathematics, with explanations.
- ☑ Parents who are concerned about their child's lack of enjoyment and success with mathematics.
- ☑ **Engagement-oriented** teachers looking for mathematics teaching resources that concern developing understanding and proficiency.

WHAT ARE THE LEARNING GOALS?

- ☑ To understand the meaning of addition as representing a '**put together**' action or a '**part/whole**' static situation.
- ☑ To be able to detect addition in a problem scenario and to solve the problem.
- ☑ To gain power in problem solving involving an **arithmetic operation**. This is sometimes known as becoming competent with **social utility problem solving**.

Why Develop Meaning First?

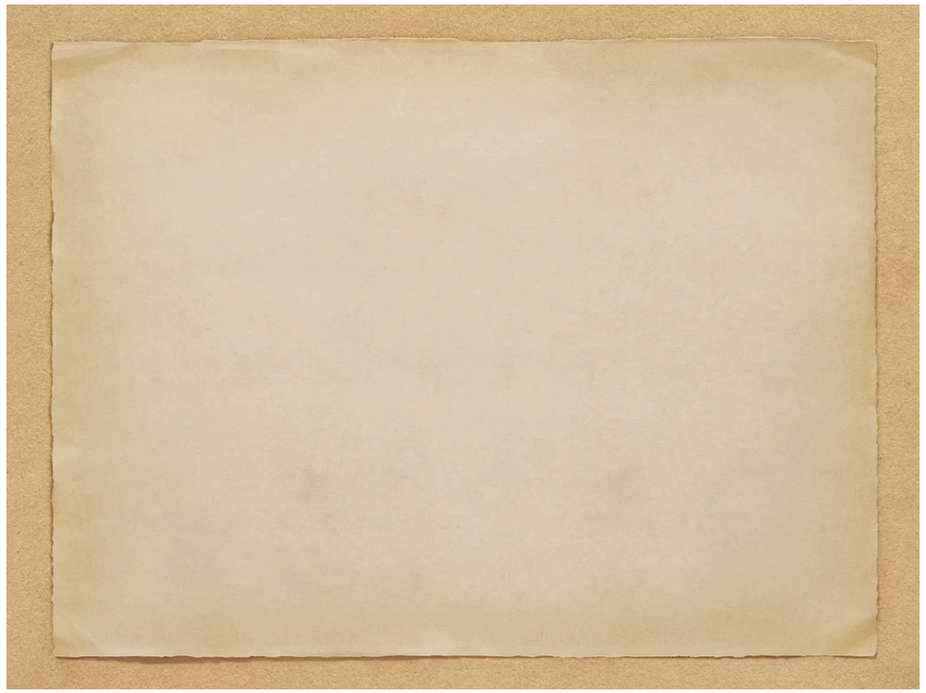
DEVELOPING MEANING

The meaning of addition is developed before arithmetic skills are developed.

WHY?

The movie provides an explanation.

Click/tap the movie.



Learning Readiness

SKILLS & CONCEPTS

- Say the number words from zero to at least twenty (**nursery rhyme counting**).
- Count a collection of at least 20 objects (**rational/real counting**).
- Count forwards by one more. [e.g.: What comes after 5? Response: 6.]
- Write the numbers from 0 to at least 20.
- zero - a count of a collection that is empty (has no objects in it).
- **parallel counting**.

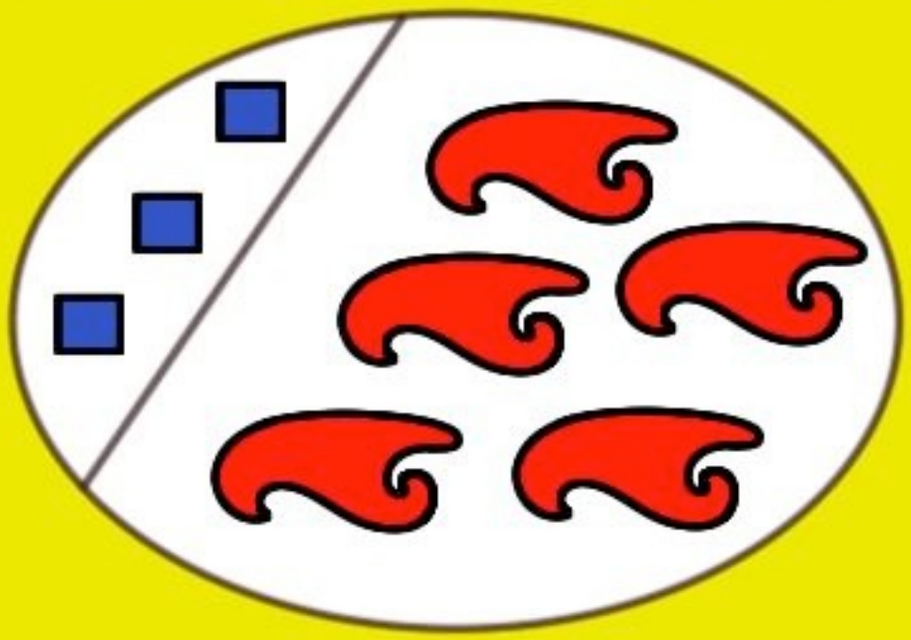
Chapter 2

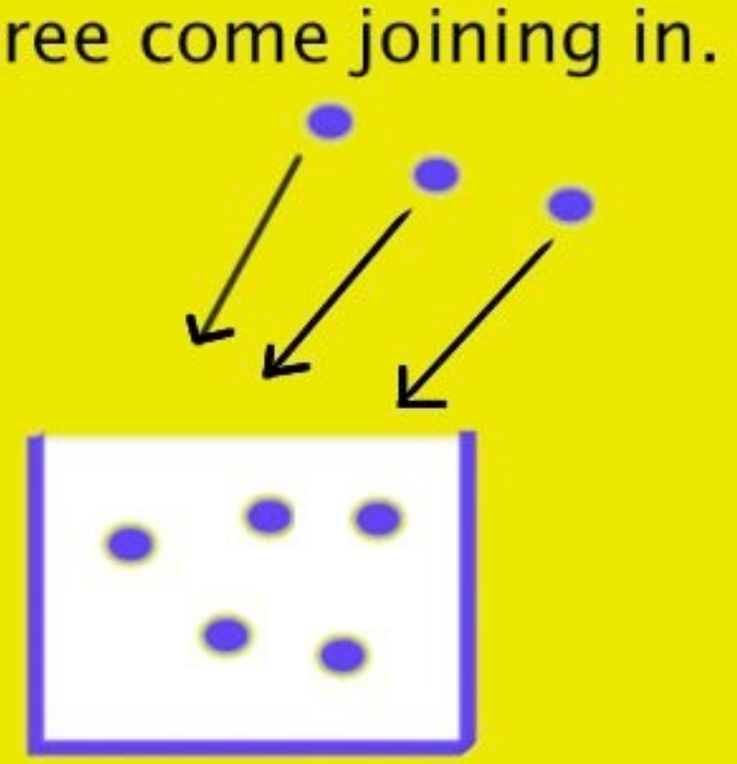
Lesson 1

Develop the meaning of addition:

(1) The part/whole sense

(2) The put together sense





Lesson 1: Overview

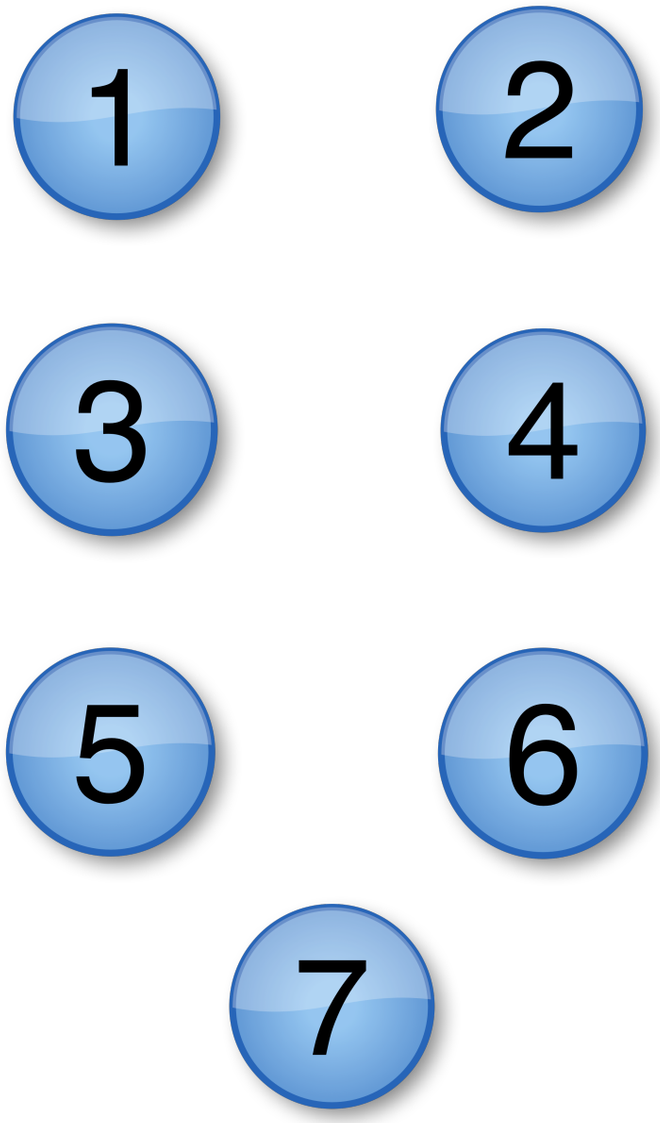
LESSON 1 ACTIVITIES

1. *Reading a story*
2. *Combining objects*
3. *Number line model*
4. *Practice*
5. *Part/whole*
6. *Practice*
7. *Assessment of teaching*

Time to complete:

About 100 minutes (10-20 minute sessions).

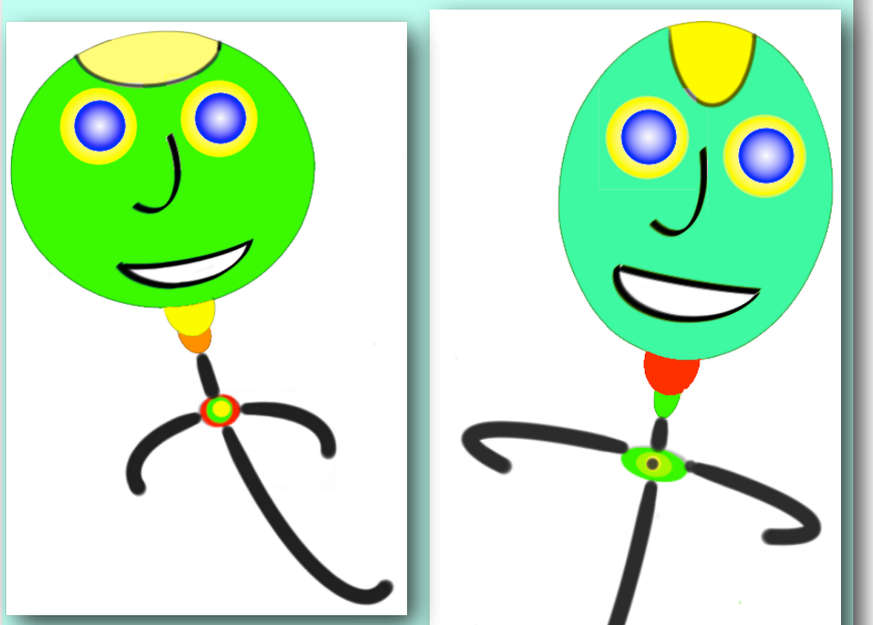
Click/tap a Number



Lesson 1: Activity 1

READING A STORY

Click Me



In the land of Moreof, live Skee and Tollof.

No legs to go; glide they do so.

Hooks they have for holding on to nooks.

High into the sky they fly.

Adventures they like, ever so bright.

continued . . .

Lesson 1: Activity 1

Aunt Zall's place of hiding is the Forest of Sliding.

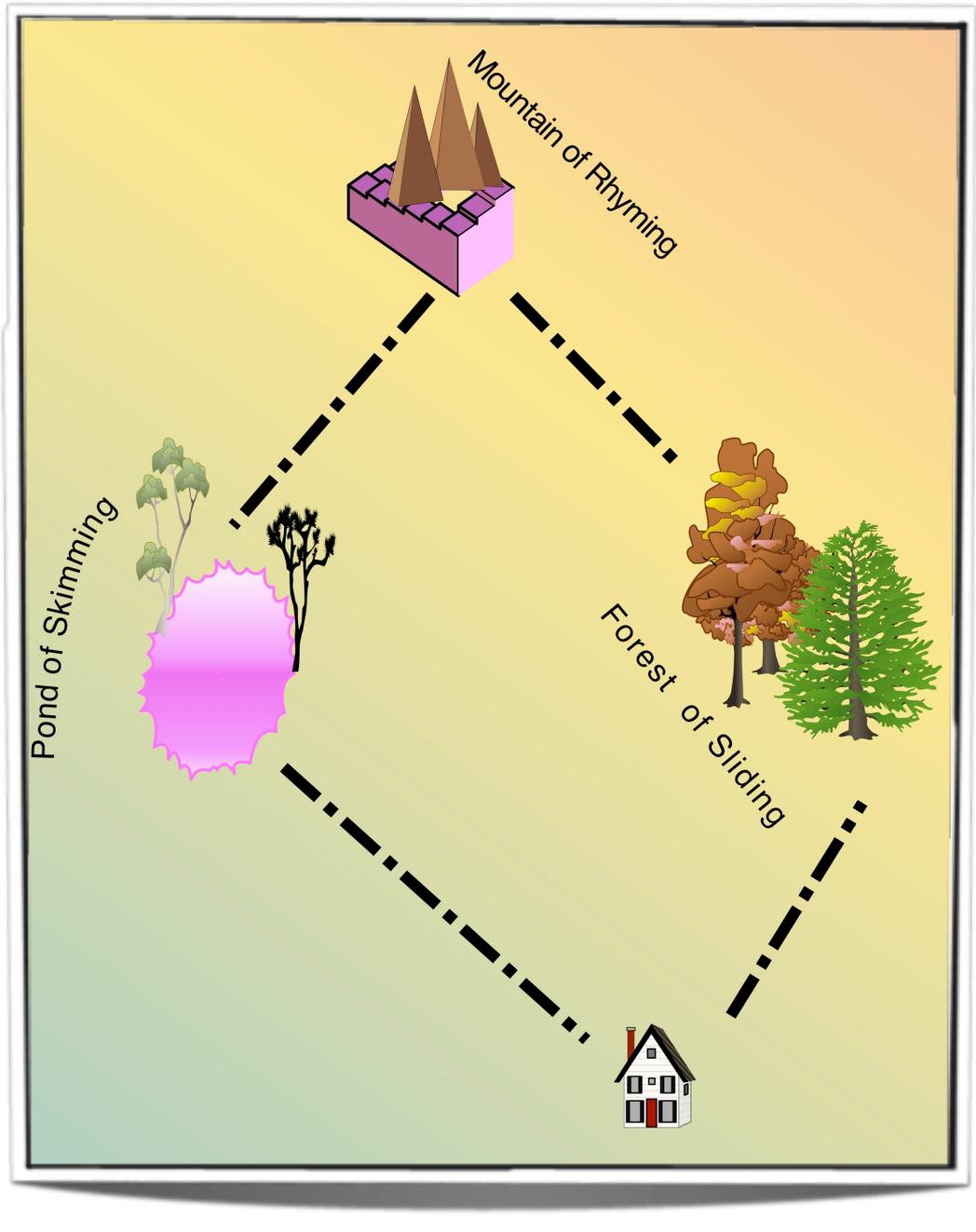
Uncle Zedd's place of climbing is the Mountain of Rhyming.

Cousin Zorr's place of swimming is the Pond of Skimming.

To Aunt Zall, they go on wings so small.

The path is long but not wrong.

continued . . .



Lesson 1: Activity 1

Hugs galore are at the door.

One is here and more appear.

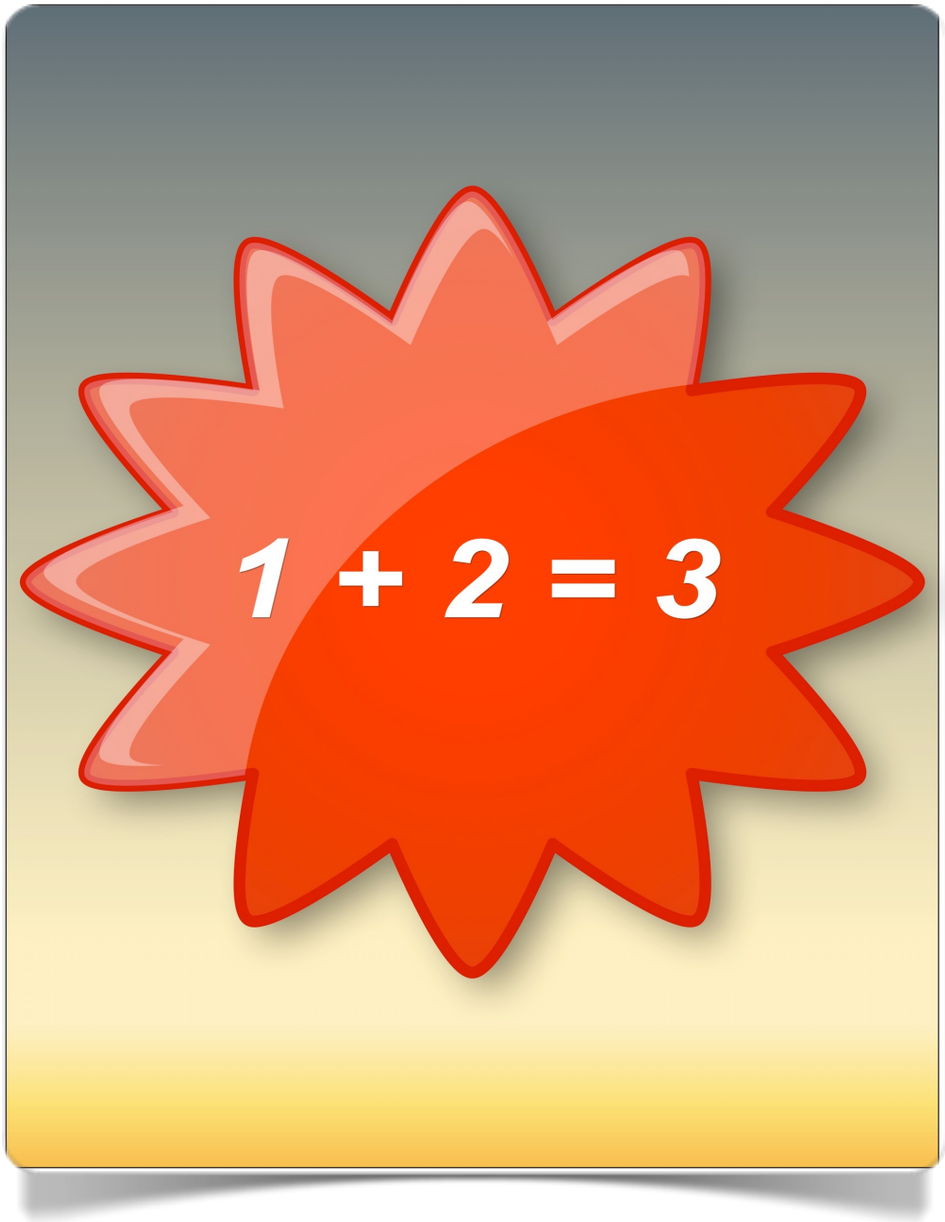
A sign is there for them to stare.

What does it mean?

What does it mean?

Oh dear! Oh dear!

Click Me



Time to go; time to go

Uncle Zedd is out of bed.

continued . . .

Lesson 1: Activity 3

NUMBER LINE MODEL

(a) Discuss Skee and Tollof visiting Uncle Zedd. Ask what '3 + 5 = 8' might mean in relation to the story. [Expect something like: Climb 3 steps and 5 steps. 8 in all]

(b) Show the number sentence '3 + 2 = 5'. Ask what the number sentence might mean in relation to the story. [Expect something like: Climb 3 steps, then 2 steps, 5 in all]

(c) Provide a number line. Discuss how it is like steps. Ask the student to show '3 + 2 = 5' on the number line. Assist as needed. Repeat for a different number sentence. [Use numbers less than 10.]

Click Me

(d) Show the number line model for '4 + 5 = 9'. Ask the student to write the number sentence shown on the number line. Repeat for a different number line model.

(e) Tell a simple story about steps (e.g. A grasshopper hopped up 4 steps, then up 5 steps. Hopped up 9 steps in all). Ask the student to show the story on the number line and to write the number sentence. Repeat for a different story.

(f) Show the number line model for '5 + 1 = 6'. Ask the student to write the number sentence shown on the number line and to tell a climbing story for it. Repeat for a different number line.

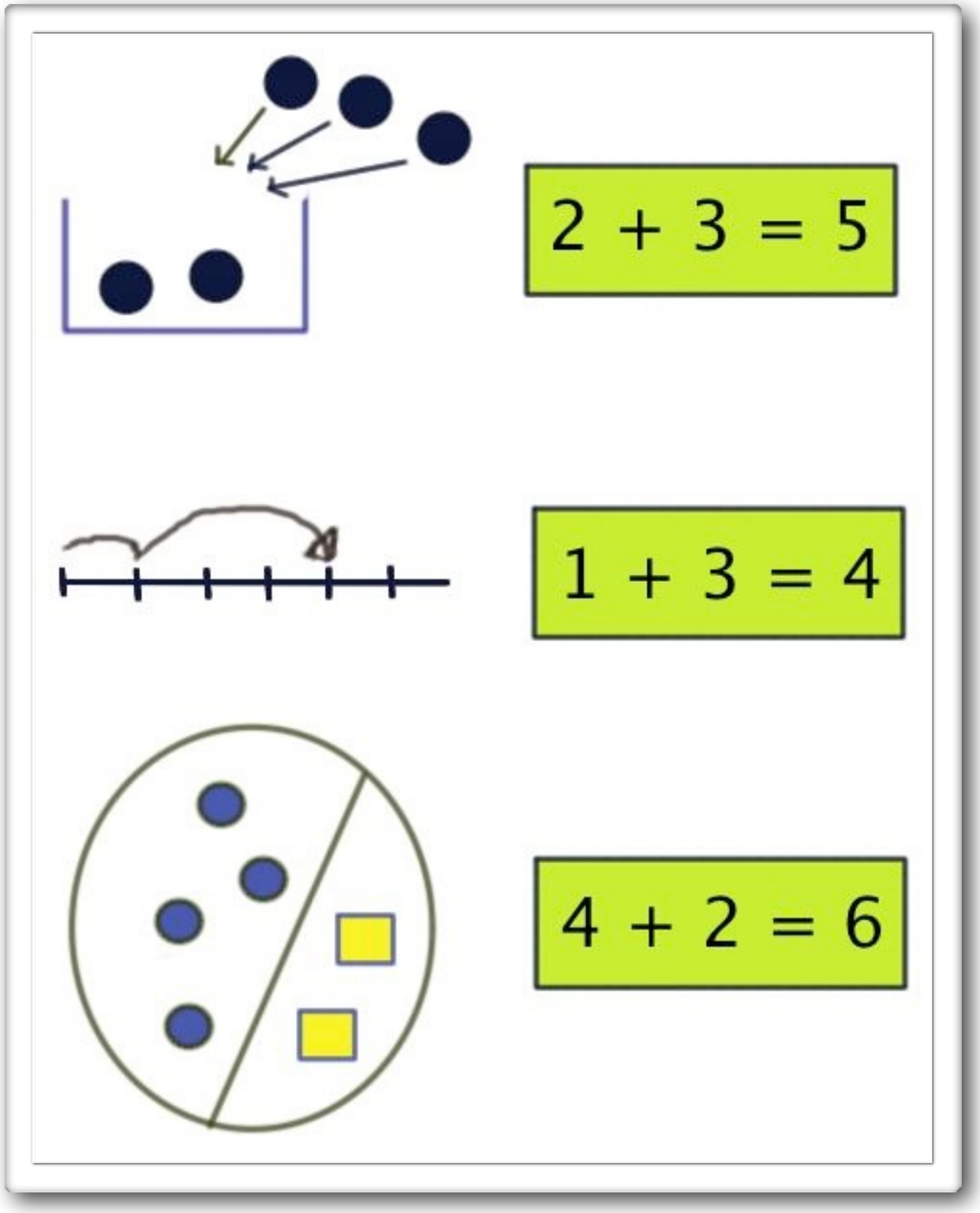
Lesson 1: Activity 6

PRACTICE

(a) Show the number sentence '3 + 5 = 8'. Ask the student to tell what it means in relation to: (1) the visit to the Forest of Sliding, (2) the visit to the Mountain of Rhyming, and (3) the visit to the Pond of Skimming.

Click Me

(b) Show the student the diagrams on this page BUT <u>cover up the number sentences</u>. Ask him/her to say/write the number sentence for each diagram. Discuss his/her responses.



Chapter 3

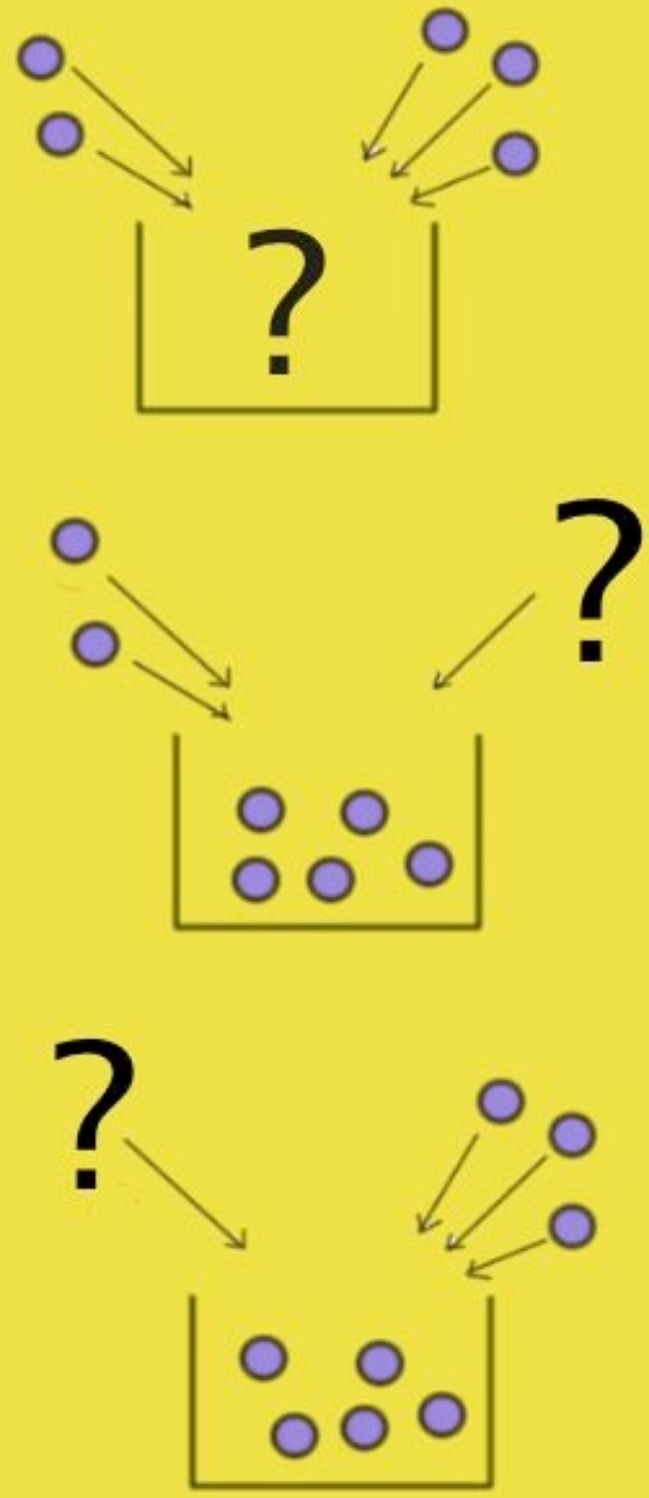
Lesson 2

Develop problem solving for:

(1) a + b = ?

(2) a + ? = c

(3) ? + b = c



Lesson 2: Overview

LESSON 2 ACTIVITIES

1. *a + b = ___ Story problem*
2. *a + b = ___ 'Put together'*
3. *a + b = ___ 'Part/whole'*
4. *a + b = ___ Practice & ten-frames*
5. *a + ___ = c Story problem*
6. *a + ___ = c Practice on number line*
7. *a + ___ = c Practice with 'put together'*
8. *___ + b = c Story problem*
9. *___ + b = c 'Put together'*
10. *Practice (put together): a + b = ___; a + ___ = c; ___ + b = c*
11. *Practice (number line): a + b = ___; a + ___ = c; ___ + b = c*
12. *Practice (stories): a + b = ___; a + ___ = c; ___ + b = c*
13. *Assessment of teaching*

Time to complete:
About 180 minutes (10-20 minute sessions).

Click/tap a Number

1 2 3 4

5 6 7 8

9 10 11 12

13

Lesson 2: Activity 1

'A + B = __' STORY PROBLEM

(a) Read the story problem.

Click Me

(b) Ask the student to write the number sentence for the problem, using a blank to indicate what is not known. [Expect: 5 + 2 = ___]

(c) Ask the student to explain why the problem is about adding. [Expect some sense of combining in the explanation.]

(d) Ask the student to use counters to solve the problem and to write the answer in the appropriate place (the blank) in the number sentence. [Ensure begins with 5, puts in 2, then counts up what is there.]

Aunt Zall says to all.

Five at the zoo and coming are two.

How many then?

How many then?

Oh dear! Oh dear!

Lesson 2: Activity 10

PRACTICE ('PUT TOGETHER')

In turn, provide one put together diagram for 'a + b = ___', one for 'a + ___ = c', and one for '___ + b = c'. [Refer to examples.]

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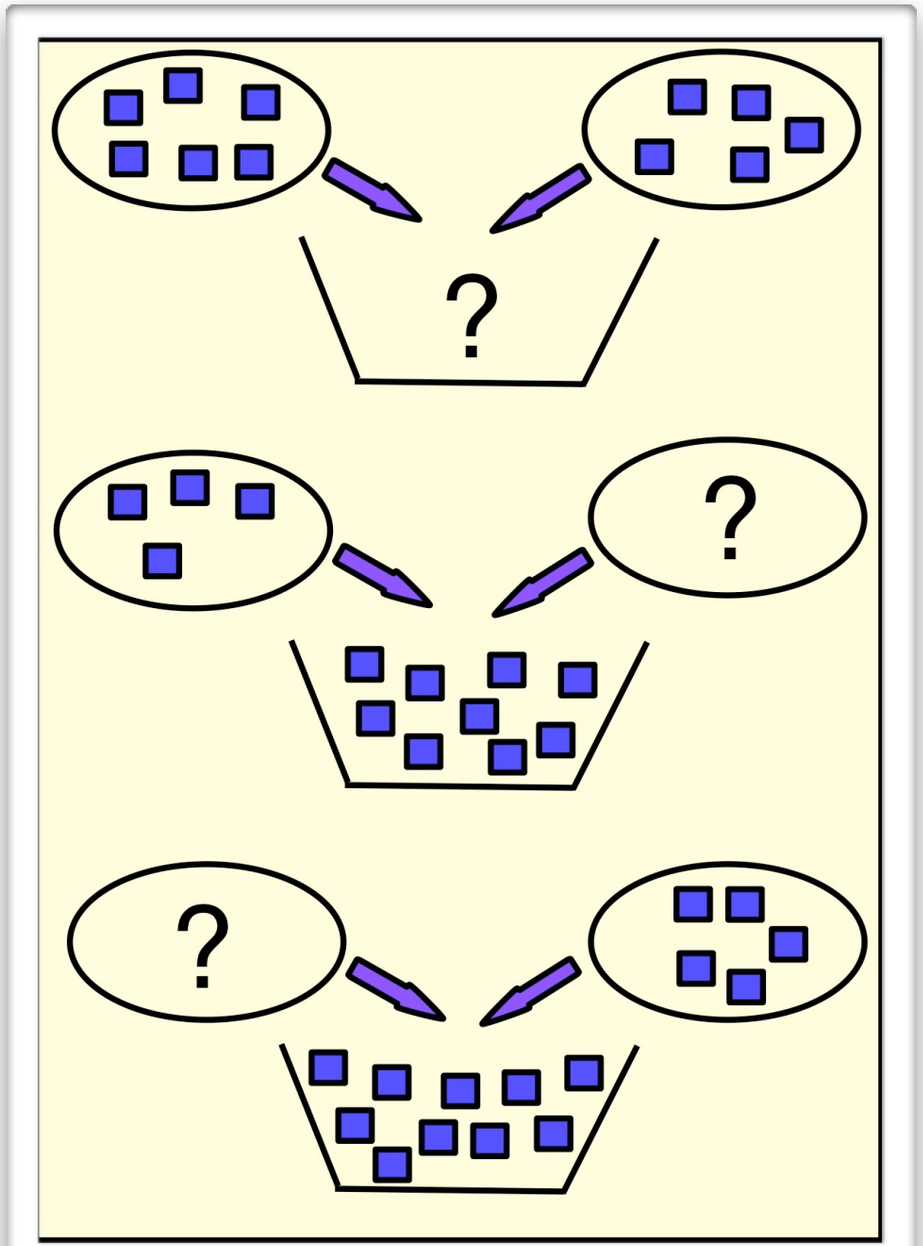
For each diagram, ask the student to:

- tell a short story problem that involves combining things.
- write the number sentence.
- obtain the solution by counting objects.



Chapter 4

iPad Practice

Student Practice for:

(1) a + b = ?

(2) a + ? = c

(3) ? + b = c

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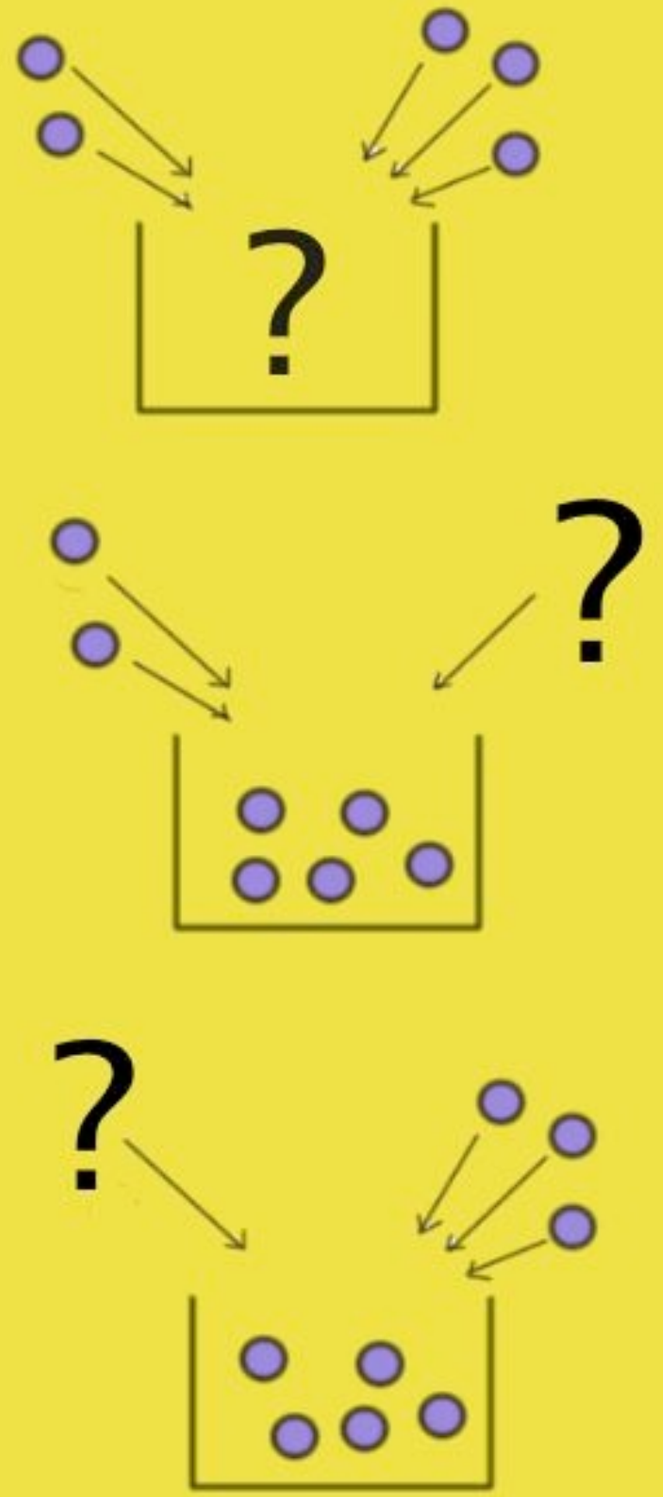


Student Practice

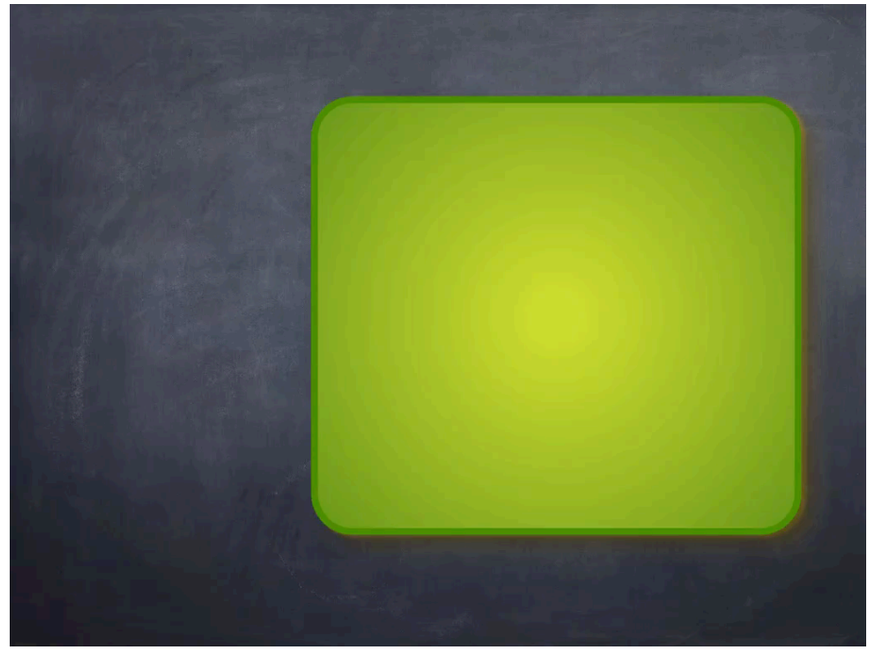
WHICH ME ?

ME ? 2 + 3 = ___

ME ? 2 + ___ = 3

ME ? 3 + 2 = ___

Click/tap the movie.



Student Practice

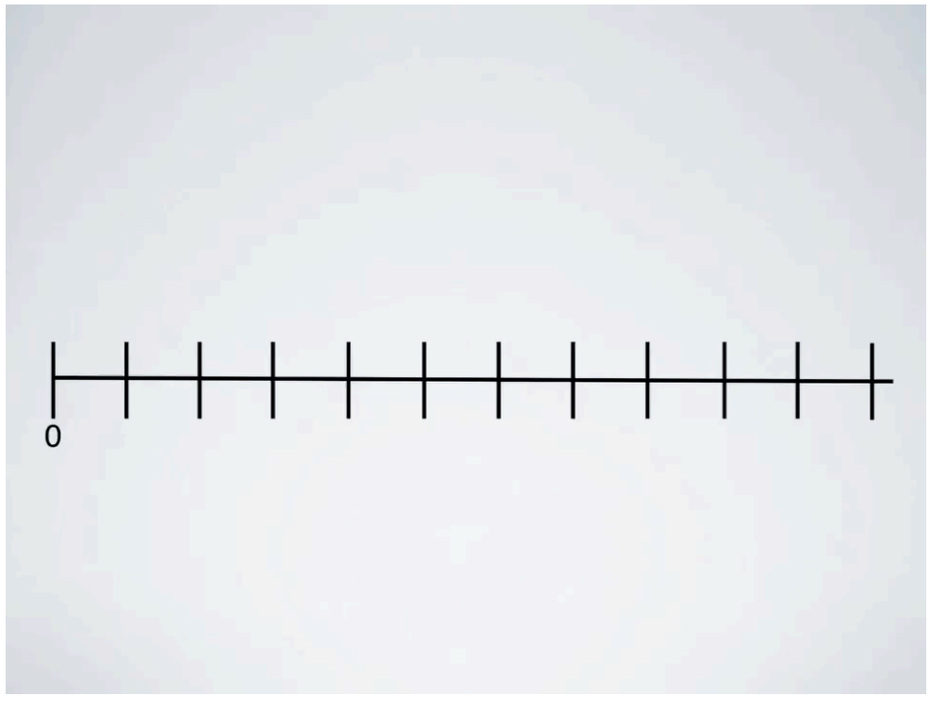
WHICH ME ?

ME ? ___ + 7 = 3

ME ? 3 + ___ = 7

ME ? 3 + 7 = ___

Click/tap the movie.



Arithmetic operation

Arithmetic operations were invented to model/represent real world situations in a symbolic, manipulable way. This is why understanding the meanings of the operations is critical to successful social utility problem solving.

Related Glossary Terms

Number sentence, Part/whole, Put together, Social Utility problem solving

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Index Find Term

Chapter 1 - Purpose

Engagement-oriented

This involves setting up a problem-solving climate of learning. The key indicators are:

- ☑ The teacher posing questions that stimulate student thinking about what is being learned.
- ☑ The teacher encouraging students to figure things out on their own. This transfers some of the ownership of learning to students.

This style of teaching is more likely to develop student understanding and interest in what is being learned.

This teaching style is dramatically different from a SHOW-AND-TELL style where the teacher acts as the teller of what is to be learned. This style is dominated by teacher demonstration and student practice of what was demonstrated. It tends not to foster student engagement and thinking. Rather it tends to foster a student attitude of '**Show me what to do and I'll give it back to you to prove I learned it. Don't ask me if I understand any of it though.**'

Related Glossary Terms

Drag related terms here

Index Find Term

Chapter 1 - Purpose

Number sentence

Mathematics is a language. It communicates meaning and has rules, just like any natural language such as English, French, Spanish, Cree . . .

When working with K–4 students, a mathematical object such as '2 + 3 = 5' is best referred to as a number sentence rather than as an equation (formal mathematical jargon). The term 'number sentence' supports the understanding of mathematics as a language and parallels the term 'sentence' that children encounter when learning English or French or . . .

Related Glossary Terms

Arithmetic operation, Part/whole, Put together, Social Utility problem solving

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Index Find Term

Nursery rhyme counting

Knowing the counting words (e.g. one , two, three . . .) by “heart” but not understanding what they refer to/mean.

Related Glossary Terms

Drag related terms here

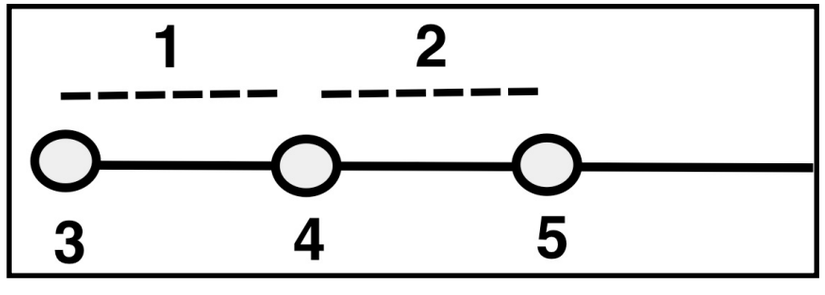
Index Find Term

Chapter 1 - Learning Readiness

Parallel counting

Being able to keep track of two counting chains (result chain and add on chain) at the same time and understanding that you start the add on chain at the next number.

For example, if you are at '3' and want to count two more, the result chain is 4, 5. The add on chain is 1, 2.



Not understanding this is sometimes referred to as the "Monopoly" oops. Imagine the milk bottle is at Park Place. The dice indicate to move 7 spaces forward. People who have the Monopoly problem will begin the count of 7 at Park Place rather than at the next square, Luxury Tax.

Related Glossary Terms

Rational/real counting

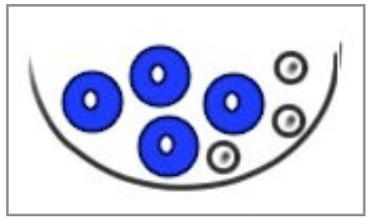
Index Find Term

Chapter 1 - Learning Readiness

Part/whole

Addition can represent a 'part/whole' situation. Consider the following situation.

Sally keeps all of her buttons in a cup. There are 4 large buttons and 3 small buttons in it. In all, there are 7 buttons in the cup.



The whole is the contents of the cup (7 buttons). The parts are the large buttons (4) and the small buttons (3). This part/whole situation can represented by an addition number sentence: '4 + 3 = 7'. There is no action involved. At some point in the past, the buttons were placed in the basket. That action is not part of the description of the situation.

The template for a part/whole situation can be thought of as:

part 1 + part 2 = the whole

The order of adding does not matter for the part/whole meaning because there is no explicit action of combining taking place. Either part can be thought of as part 1.

Related Glossary Terms

Arithmetic operation, Number sentence, Social Utility problem solving

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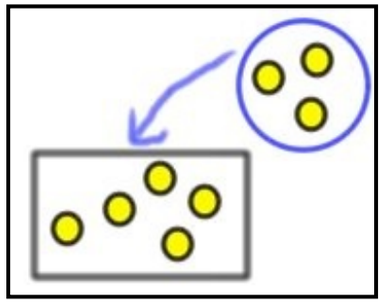
Index Find Term

Chapter 1 - Purpose

Put together



One sense of addition concerns combining – an explicit 'put together' action. Consider the diagram. The rectangle represents "all – the pot" and the circle represents what is being combined (put together) with what is already in the "pot".

The number sentence, '5 + 3 = 8', models the situation mathematically. Each symbol in the number sentence has a particular role.

- '5' is the count of what is already in the "pot".
- '+' indicates a combining (put together) action.
- '3' is the count of what is being placed into the "pot".
- '=' is the signal to count up what is in the "pot" after the combining action is over.
- '8' is the count of what is in the "pot" after the action is over.

The role of '=' is not algebraic (what is on the left is another name for what is on the right). Rather, '=' is a one-way arrow that tells us to count what is finally there. The algebraic role of '=' becomes important when doing arithmetic and algebra. Then it acts as a two-way arrow; what is on the left side is another way of saying what is on the right side.

Related Glossary Terms

Arithmetic operation, Number sentence, Social Utility problem solving

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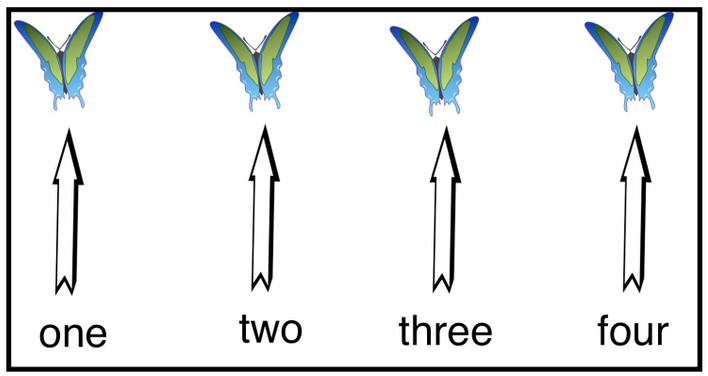
Index Find Term

Chapter 1 - Purpose

Rational/real counting

A count tells you how many objects in a collection of objects.



Rational counting involves matching a number word to an object. The last number word spoken (e.g. four) represents the count of the collection of objects. This is related to, but different from, nursery rhyme counting in which the counting words are not matched to objects.

Related Glossary Terms

Parallel counting

Index Find Term

Chapter 1 - Learning Readiness

Social Utility problem solving

A social utility problem involves a real world situation that can be represented by an arithmetic operation(s). The first, and critical, step is deciding what arithmetic operation to use. The second step is doing the arithmetic in some way.

Here is a sample problem given to grades 4 and 5 students.

Harry went on a trip to the Nile River to collect crocodile eggs. He soon found a crocodile nest. He dug up 23 eggs and placed them in a basket. Harry walked 5 paces to another nest and dug up all the eggs he found there and placed them in the basket. Harry's basket now had 40 crocodile eggs in it. How many eggs did Harry find at the second crocodile nest?

The problem involves 'put together'. That is represented mathematically by the number sentence; '23 + ? = 40'. [Start with 23 eggs, put in an unknown amount, end up with 40 eggs.]

Edited responses of three students:

Student #1

The student read the problem carefully, underlined all the printed numbers, and said "Easy numbers!". The answer that the student gave (44) was incorrect (but strongly insisted that it was correct). The student's work consisted of three steps: (a) 40 + 5 = 45, (b) 45 - 23 = 22, and (c) 22 x 2 = 44. To explain the work, the student said that since there were forty eggs, it was necessary to add the little number ('5'), and then to subtract 23. That answer had to be multiplied by 2 because the word 'second' was in the problem.

Student #2

The student did not do the problem. The student explained that the problem did not make any sense because it did not fit the problem-solving strategies (guess & check and look for a pattern) learned in class.

Student #3

The student made a guess & check chart, but did not do any guessing. The student obtained the correct solution by subtracting 23 from 40. The student could not explain why. When asked if guessing was done, the reply was "No." When asked why the guess chart was made, the student said; "All word problems are done by guess & check."

Discussion

The three responses expose some issues with respect to social utility problem solving. The lack of success in solving such problems can be explained by the strategies that students use. Here is a partial list.

- Find the numbers and add (or multiply or . . .). The choice is dictated by recent activities or by what the child is comfortable with.
- Guess the operation to be used and see what you get. This is a form of 'guess & check'.
- Look at the numbers; they will "tell" you which operation to use (for example, the numbers 63 and 59 suggest add or subtract, while 25 and 5 suggest divide).
- Try all the operations and select the most reasonable answer.
- Look for key words which tell you what to do.
- Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication, and select the more reasonable answer. If smaller, try both subtraction and division and select the more reasonable answer.
- Choose the operation whose meaning fits what is happening in the problem.

The strategy 'choosing the operation whose meaning fits the story' will lead to more success in social utility problem solving. It involves identifying what is going on in the problem and then representing that with a number sentence.

The story problem was about combining eggs. The action is represented by '23 + ? = 40'. To obtain the answer, you can alter the number sentence and use subtraction (40 - 23 = ?) or you can add up from 23 (e.g. +7 yields 30; then +10 yields 40; 7 + 10 is 17).

Related Glossary Terms

Arithmetic operation, Number sentence, Part/whole, Put together

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Index Find Term

Chapter 1 - Purpose